Final Exam

Your Name:

Instructions

Solve each of the following problems to the best of your abilities. The exam is worth 100 points total and is calibrated for 120 minutes. Once you have completed the exam, hand it to me, and you can take a break before lab. Lab starts at 7:30PM.

Each question is worth four points. Good luck!

1. Two point charges, each with a charge of +1.5 mC, are separated by a distance of 0.20 m. What is the electric force between the particles? Do they attract or repel?

$$F = \frac{kq_1q_2}{r^2} = \frac{\left(9 \times 10^9 \frac{Nm^2}{C^2}\right)(1.5 \times 10^{-3}\ C)^2}{(0.20\ m)^2} = 506250\ N$$

They repel each other since they are the same sign.

2. Sketch a diagram showing some electric field lines and equipotential lines around a single positive point charge. Be sure to label the field lines and the equipotential lines.

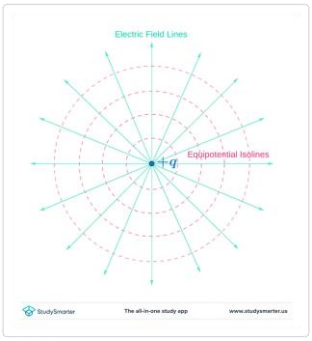

Image courtesy of https://www.vaia.com/en-us/explanations/physics/electric-charge-field-and-potential/equipotential-lines/

3. Two point charges, each with a charge of +1.3 mC, are separated by a distance of 0.15 m. What is the electric potential energy of the two particle system?

$$U = \frac{kq_1q_2}{r} = \frac{\left(9 \times 10^9 \frac{Nm^2}{C^2}\right)(1.3 \times 10^{-3}\ C)^2}{0.15\ m} = 101400\ N$$

4. A chunk of material stores a charge of 2.70 C when a voltage of 9 V is connected to it. What is its capacitance?

$$C = \frac{Q}{V} = \frac{2.70\ C}{9\ V} = 0.30\ F$$

5. Why might we use a dielectric material within a parallel plate capacitor?

A dielectric material reduces the electric field between the plates of the capacitor, allowing a capacitor to store more charge.

6. Four resistors, each with a resistance of 20 Ω, are connected in series along with a battery of voltage 10 V. What is the current drawn from the battery?

$$I = \frac{V}{R_{eq}} = \frac{10\ V}{4(20\ \Omega)} = 0.125\ A$$

7. A positively charged particle is fired into a magnetic field with an initial velocity in the +x direction. If the magnetic field is pointing in the –y direction, in which direction is the initial force on the particle?

The force is in the –z direction, as per the right hand rules.

8. A velocity selector consists of a crossed electric field (1000 N/C) and magnetic field (0.5 T). What speed should I fire a positively charged particle through the velocity selector if I want it to pass through, undeflected?

$$v = \frac{E}{B} = \frac{1000\ \frac{N}{C}}{0.5\ T} = 2000\ \frac{m}{s}$$

9. A charging DC LR circuit consists of a battery (10 V), a resistor (100 Ω), and an inductor (0.5 H) in series. What is the time constant of the circuit?

$$\tau = \frac{L}{R} = \frac{0.5\ H}{100\ \Omega} = 0.005\ s$$

10. The magnetic flux passing through a loop of wire is given by $\phi_B(t) = 10\,t^2\ Wb$. What is the induced EMF in the loop at time t = 2 seconds (including the correct sign)?

$$\epsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(10t^2) = -20t$$

$$\epsilon(2\ s) = -20(2\ s) = -40\ V$$

11. A sinusoidal signal has a peak voltage of 10 V. What is the RMS voltage of the signal?

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{10\ V}{\sqrt{2}} = 7.07\ V$$

12. An AC RLC circuit is made up of a 10 ohm resistor, a 1.5 mF capacitor, and a 1.8 mH inductor. What is the phase angle of the circuit?

I should have given you a frequency for the circuit in the problem statement – otherwise, you cannot answer the question. Since I messed up this question, everybody gets full points for it.

13. In the context of the Ampere-Maxwell equation, how is displacement current different from “regular” current?

Displacement current is the changing electric flux through an area whereas current is charge moving past an area.

14. Modern microwave ovens operate at the frequency 2,450 MHz. What is the wavelength of these waves?

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \frac{m}{s}}{2450 \times 10^6 \, Hz} = 0.112 \, m$$

15. Light, initially polarized along the z-axis, encounters a polarizing filter oriented at 25 degrees with respect to the z-axis. What percentage of light passes through the filter?

$$\frac{I_2}{I_1} = \cos^2(25°) = 0.82$$

16. A supernova is an exploding star that is billions of times brighter than an ordinary star. Supernova SN1987a occurred 1.6×10^{21} m from Earth. How long did it take light to travel from the supernova to Earth?

$$t = \frac{d}{c} = \frac{1.6 \times 10^{21} \, m}{3 \times 10^8 \, \frac{m}{s}} = 5.3 \times 10^{12} \, s$$

17. A ray of light with a wavelength of 650 nm passes from air (n = 1.00) to water (n = 1.33). Calculate the angle of reflection if the angle of incidence of the ray of light is 21 degrees.

Angle of reflection = Angle of incidence = 21 degrees

18. A ray of light with a wavelength of 650 nm passes from air (n = 1.00) to water (n = 1.33). Calculate the angle of refraction if the angle of incidence of the ray of light is 45 degrees.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \;\Rightarrow\; (1.00)\sin(45°) = (1.33)\sin\theta \;\Rightarrow\; \theta = 32°$$

19. What is the critical angle as light travels from plate glass (n = 1.52) to benzene (n = 1.501)?

$$\theta_c = a\sin\left(\frac{1.501}{1.52}\right) = 80.9°$$

20. What does wave-particle duality mean in the context of light?

Wave particle duality tells us that light has an interesting and weird property that sometimes it acts like a wave, while other times it acts like a particle.

21. An object with a height of 5 cm is located 20 cm in front of a converging mirror with a focal length of |f| = 10 cm. Where is the image located?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \;\Rightarrow\; \frac{1}{+10 \, cm} = \frac{1}{+20 \, cm} + \frac{1}{s'} \;\Rightarrow\; s' = 20 \, cm$$

22. An object with a height of 10 cm is located 30 cm in front of a diverging lens with a focal length of |f| = 5 cm. What is the final height of the image?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \;\Rightarrow\; \frac{1}{-5\ cm} = \frac{1}{+30\ cm} + \frac{1}{s'} \;\Rightarrow\; s' = -4.29\ cm$$

$$M = -\frac{s'}{s} = -\frac{-4.29\ cm}{30\ cm} = +0.143$$

$$h' = 0.143\,h = 1.43\ cm$$

23. The index of refraction of fused silica is around 1.51. What is the wavelength of red light (700 nm in vacuum) as it passes through the material?

$$\lambda = \frac{\lambda_o}{n} = \frac{700\ nm}{1.51} = 463.6\ nm$$

24. A double slit interference apparatus has a slit separation of 0.2 mm. At what angle would we see the 20th order maximum if we shine light with a wavelength of 600 nm on it?

$$d \sin\theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(20)(600\ nm)}{0.2\ mm}\right) = 3.43^\circ$$

25. A thin film of soapy water (n = 1.33) is sandwiched between layers of air in a soap bubble. At what minimum thickness of the film would I see red light with a wavelength of 700 nm?

https://www.youtube.com/watch?v=lpSGC57nsg0

26. (Optional Bonus) In class, we went through a derivation of the equation for electromagnetic waves, starting from Maxwell's equations. What do we mean by the "curl" and "divergence" of a vector field? You do not have to show any mathematics – an explanation will suffice.

https://www.youtube.com/watch?v=rB83DpBJQsE