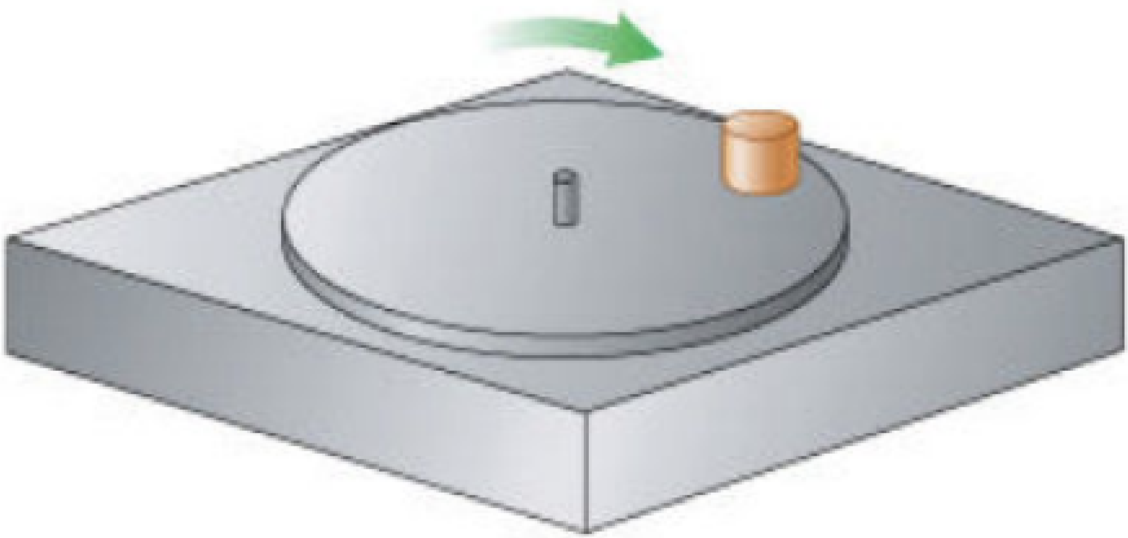
Exam 2 Solutions

Instructions

- Write your name on this test booklet.
- You are allowed a writing utensil, a calculator, and the PHY 201 formula sheet.
- You will have 110 minutes to complete this exam.
- Good luck!

Problem 1

(25 points) That fly from exam one lands on a turntable that is initially at rest. It is sitting at a distance of 0.10 m from the center of the disc. The turntable spins up with an angular acceleration of $5.0\ rad/s^2$ for 3 seconds and then continues with a constant angular velocity.



1. What is the angular speed of the fly and the turntable in units of radians per second after the initial acceleration?

$$\vec{\omega}_f = \vec{\omega}_o + \vec{\alpha}t = (0\ rad/s) + (5.0\ rad/s^2)(3\ s) = 15 rad/s$$

2. What is the angular speed of the fly and the turntable in units of revolutions per second after the initial acceleration?

$$\left(\frac{15\ radians}{1\ second}\right)\left(\frac{1\ revolution}{2\pi\ radians}\right) \approx 2.39\ rev/s$$

3. Friction keeps the fly on the turntable as it rotates. What is the coefficient of static friction between the fly and the turntable?

Let's pretend the mass of the fly is m. Then:

$$F_{centripetal} = mr\omega^2$$

$$F_{friction} = \mu N = \mu mg$$

$$mr\omega^2 = \mu mg \Rightarrow \mu = \frac{r\omega^2}{g} = \frac{(0.10\,m)(15\,rad/s)^2}{9.8\,m/s^2} \approx 2.30$$

Yeah, this does seem weird that the coefficient of friction is larger than one. But there is no "law of nature" that says the coefficient of friction has to be less than one. It just means that it is a very sticky turntable.

4. How would the centripetal acceleration on the fly change if it moves closer to the center of the turntable? Why?

If you moved the fly closer to the center of the turntable, the radius, r would decrease. Since $a_c = r\omega^2$, the centripetal acceleration would also decrease.

Some people said that $a_c = v^2/r$, so the centripetal acceleration should increase. Remember that $v = r\omega$. Thus, both the numerator and denominator will decrease in this expression, but the numerator will decrease at a faster rate. Thus, the overall centripetal acceleration will decrease.

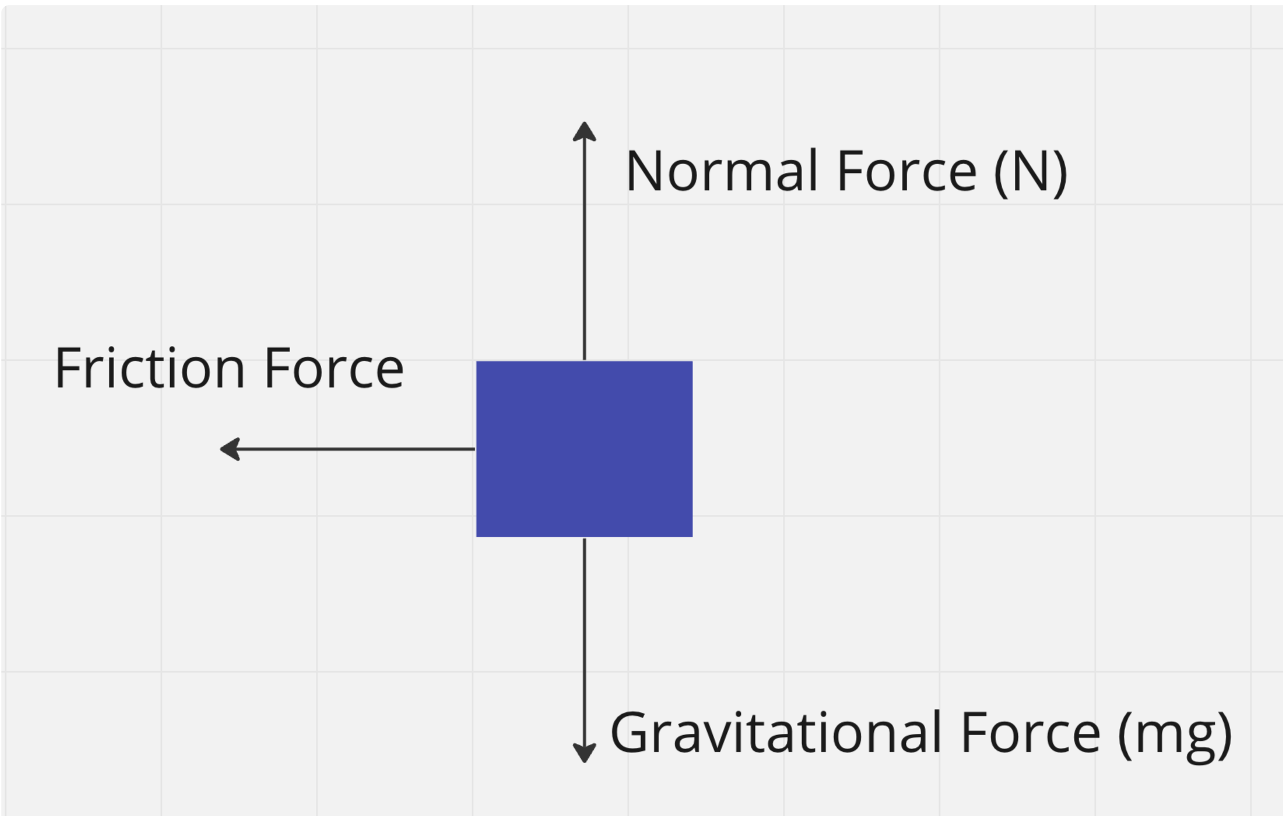
5. How would the centripetal acceleration on the fly change if I increased the mass of the fly? Why?

The centripetal acceleration of an object is given by $a_c = r\omega^2 = v^2/r$. Since there is no mass term, the mass of the fly makes not difference. The centripetal acceleration would not change.

Problem 2

(25 points) A block with a mass of $m = 15$ kg is brought up to an initial velocity $v_o = 13$ m/s along the floor and then left to slide without any additional pushing. It slides for a distance of 10 meters before coming to a complete stop due to friction.

1. Draw a free-body diagram of the block while it is sliding along the floor (without any additional pushing).



Notice how there is no force due to pushing the block - this FBD describes while the block is sliding.

2. What is the acceleration of the block over the 10 meters it slides?

Let's use kinematics:

$$v_f^2 = v_o^2 + 2a\Delta x$$

$$(0\, m/s)^2 = (13\, m/s)^2 + 2a(10\, m)$$

$$(0\, m/s)^2 = (13\, m/s)^2 + 2a(10\, m)$$

$$|a| = 8.45\, m/s$$

The acceleration is in the opposite direction to the initial velocity.

3. What is the coefficient of kinetic friction between the block and the ground?

$$\vec{F}_{friction} = \mu N\,(-\hat{x}) = \mu mg\,(-\hat{x})$$

$$\vec{F}_{net} = \mu mg\,(-\hat{x}) = ma(-\hat{x})$$

We can use the acceleration from the previous section to solve:

$$a = \mu g \Rightarrow \mu = \frac{a}{g} = \frac{8.45 m/s^2}{9.8\, m/s^2} \approx 0.862$$

4. What is the net work done by friction over the 10 meters that the block slides?

$$W = (F_{friction})(\Delta x) cos(\theta)$$

$$W = (\mu mg)(\Delta x) cos(\theta)$$

$$W = (0.862)(15\, kg)(9.8\, m/s^2)(10\, m) cos(180^\circ) = -1267 J$$

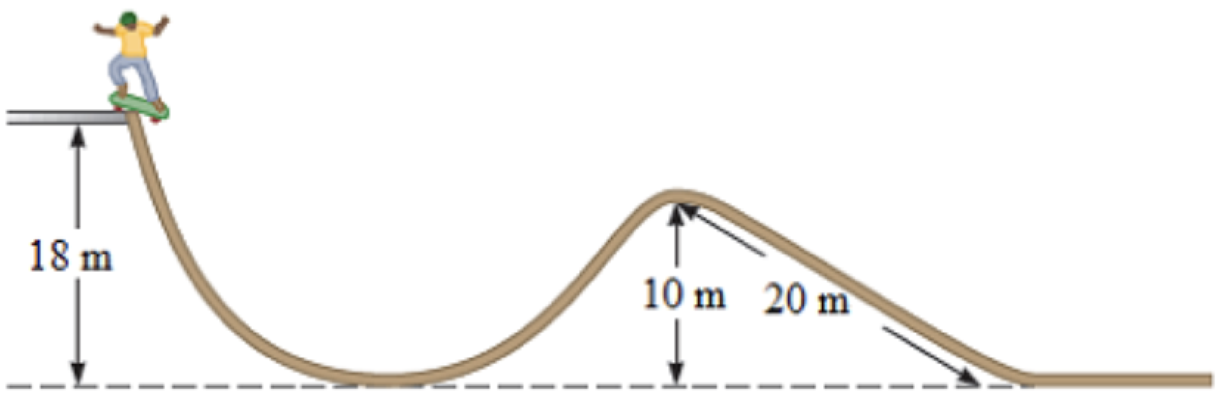
Note the negative sign! The force of friction does negative work.

5. Is friction a conservative or non-conservative force? How do you know?

It is non-conservative. Different paths will experience different amounts of friction force, even if they start and end at the same locations.

Problem 3

(25 points) A skateboarder with a mass of 65 kg rides his skateboard at a local skate park. He starts from rest at the top of the track as seen in the figure below and begins a descent down the track, always maintaining contact with the surface. The mass of the skateboard and the friction of the ramp are negligible except where noted.



1. What is the skateboarder's speed when he reaches the bottom of the initial dip, 18.0 m below the starting point?

Use conservation of energy, we can calculate the speed at the bottom of the 18 m ramp. I will let snapshot one be the skateboarder at the top of the ramp and snapshot two be the skateboarder at the bottom of the ramp (where $U_G = 0$):

$$mgh_{18} = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh_{18}} = \sqrt{(2)(9.8\,m/s^2)(18\,m)}$$

$$v = 18.8\,m/s$$

2. The skateboarder then ascends the other side of the dip to the top of a hill, 10.0 m above the ground. What is his speed when he reaches the top of the hill?

You can use the kinetic energy at the bottom of the 18 m ramp or the original gravitational potential energy to calculate the energy at the top of the 10 m hill. In my case, I am going to call snapshot one the top of the 18 m ramp, and snapshot two the top of the 10 meter ramp:

$$mgh_{18} = mgh_{10} + \frac{1}{2}mv_{10}^2$$

Canceling out mass and plugging in values gives us:

$$(9.8\,m/s^2)(18\,m) = (9.8\,m/s^2)(10m) + \frac{1}{2}v_{10}^2$$

$$v_{10} \approx\ 12.5m/s$$

3. The skateboarder then traverses the 20 m section of track where there is a coefficient of friction equal to $\mu = 0.04$. What is the final speed of the skateboarder after he clears the ramp and rides off?

Use the full conservation of energy expression, including non-conservative work. I am going to put snapshot one at the top of the 18 m hill and snapshot two after the skateboarder clears the terrain with friction. I will let v_f be the final speed of the skateboarder after the friction:

$$mgh_{18} + W_{nc} = \frac{1}{2}mv_f^2$$

The work done on the skateboarder is given by the frictional force on the skateboarder multiplied by the distance they travel multiplied by the cosine of the angle between them. Thus:

$$W_{nc} = (F_{friction})(\Delta x)cos\theta$$

The friction force is given by:

$$F_{friction} = \mu N = \mu mgcos\theta$$

You can solve for $cos\theta$ by using the lengths given in the diagram.

$$F_{friction} = \mu mgcos\theta = \mu mg\frac{\sqrt{20^2 - 10^2}}{20} = \mu mg\frac{\sqrt{300}}{20} = \mu mg\frac{\sqrt{3}}{2}$$

Then, use the force to find the work:

$$W_{nc} = (F_{friction})(\Delta x)cos\alpha$$

$$W_{nc} = (\mu mg\frac{\sqrt{3}}{2})(20m)cos(180^{\circ}) = -10\sqrt{3}\mu mg$$

Plugging everything back in, gives:

$$mgh_{18} - \mu mg\frac{\sqrt{3}}{2} = \frac{1}{2}mv_f^2$$

$$gh_{18} - \mu g\frac{\sqrt{3}}{2} = \frac{1}{2}v_f^2$$

$$2gh_{18} - \mu g\sqrt{3} = v_f^2$$

$$v_f = \sqrt{2gh_{18} - \mu g\sqrt{3}}$$

$$v_f = \sqrt{2(9.8\,m/s^2)(18\,m) - (0.04)(9.8\,m/s^2)\sqrt{3}}$$

$$v_f = 18.76m/s$$

Just barely less than the first answer, but a noticeable difference!

4. How would your answer to part one change if you increased the mass of the skateboarder? Why?

It would not change - the mass of the skateboarder cancels out when you compare gravitational potential energy and kinetic energy.

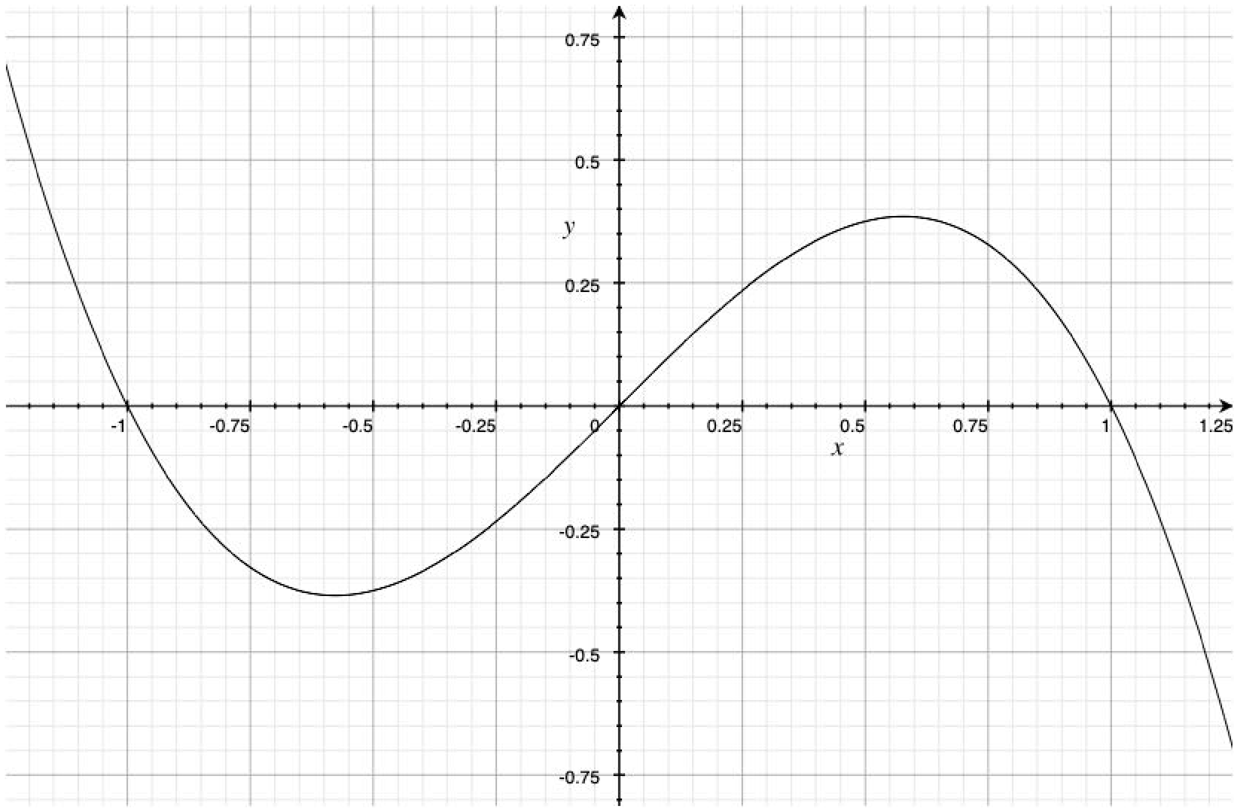
5. How would your answer to part one change if you increased the height of the initial hill from 18 meters to 36 meters? Why?

The speed of the skateboarder would increase at the bottom of the hill because they started with a larger gravitational potential energy.

Problem 4

(25 points) An object is constrained to move along the x-axis under some variable potential energy, given by $U(x) = x - x^3$. The position, x, is measured in meters and $U(x)$ is measured in Joules.

1. Sketch a graph of $U(x)$ vs x. Label the zero points, the maxima, and the minima.



To solve for the zero points:

$$0 = x - x^3 = x(1-x)(1+x) \Rightarrow x = 0, \pm 1$$

The zero points are: $(0,0)$, $(1,0)$, and $(-1,0)$.

Take the derivative and set it equal to zero to find the maximum / minimum.

Maximum occurs at: $\left(\frac{1}{\sqrt{3}}, \frac{2}{3\sqrt{3}}\right) \approx (0.577, 0.385)$

Minimum occurs at: $\left(-\frac{1}{\sqrt{3}}, -\frac{2}{3\sqrt{3}}\right) \approx (-0.577, -0.385)$

1. Find an expression for the force acting on the object as a function of position.

$$\vec{F}(x) = -\frac{dU(x)}{dx} = -\frac{d}{dx}\left[x - x^3\right] = 3x^2 - 1$$

1. What is the force acting on the object at the position $x = 1$ meter?

$$\vec{F}(1) = 3(1)^2 - 1 = 2\ N$$

1. At which position(s) is this object in equilibrium?

The object is in equilibrium when the force is equal to zero.

$$0 = 3x^2 - 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{\frac{1}{3}}$$

1. Determine if each of the equilibrium positions is a stable or unstable.

Stable equilibria occur at minimums in the potential energy graph while unstable equilibria occur at the maximums in the potential energy graph. Thus, $x = -1/\sqrt{3}$ is stable while $x = +1/\sqrt{3}$ is unstable.